RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [2018-21]
B.A./B.Sc. THIRD SEMESTER (July – December) 2019
Mid-Semester Examination, September 2019

Date : 16/09/2019 **MATHEMATICS (Honours)**
Time : 1 pm – 3 pm **Paper: III** Full Marks : 50

[Use a separate Answer Book for each group]

GROUP – A

Answer question no. 1 and any three questions from question no. 2 to 6. [2+(3×5)]

1. Find two linear operators T and U on $\mathbb{R}^3$ such that TU = 0 but UT ≠ 0.
2. U and W are two subspaces of a finite dimensional vector space V over a field F. Prove that $\dim(U+W)=\dim U+\dim W-\dim(U\cap W)$.
3. Find two different complement of the subspace U of $\mathbb{R}^4$ generated by $\{(1,2,3,4),(1,1,1,3)\}$ in $\mathbb{R}^4$.
4. V and W are two vector spaces over the field F with dimensions m and n respectively. Prove that $L(V,W)$ is finite dimensional and has dimension mn.
5. Let V be the space of $n\times 1$ matrices over F and W be the space of $m\times 1$ matrices over F and A be a fixed $m\times n$ matrix over F. Define $T:V\to W$ by $T(X)=AX$. Prove that T is a linear transformation. Also show that T is the zero transformation if and only if A is the zero matrix. [1+4]
6. Let T be the linear operator on $\mathbb{R}^3$ defined by $T(x_1,x_2,x_3)=(3x_1+x_2,\ -2x_1+x_2+x_3,\ 10x_2+3x_3)$ What is the matrix of T in the ordered basis $\{(1,2,1),(3,1,5),(2,3,1)\}$?

GROUP – B

Answer **any two** from question nos. 7 to 9 : (2 × 4)

7. Check whether the functions $f_n(x)=nx(1-x)^n, n\in\mathbb{N}.$ converges uniformly on $[0,1]$. [4]
8. Does there exist a sequence of polynomials which converges to the function $f(x)=e^x$ uniformly on $[0,\infty)$? Justify your answer. [4]
9. If $\sum u_n(x)$ is a uniformly convergent series on $[a,b]$, and $\nu:[a,b]\to\mathbb{R}$ is a bounded function on $[a,b]$, prove that $\sum_n u_n(x)\nu(x)$ is uniformly convergent on $[a,b]$. [4]

GROUP – C

Answer **any two** from question nos. 10 to 12 : (2 × 5)

10. Let *P*, *Q*, *R*, *S* be four points in space and *L*, *M*, *N*, *T* be points dividing the segments $\overline{PQ}, \overline{QR}, \overline{RS}, \overline{SP}$ in the ratios *l* : 1, *m* : 1, *n* : 1 and *t* : 1 respectively. If *L*, *M*, *N*, *T* are coplanar show that *lmnt* = 1. [5]

11. A variable line always intersects the lines $x = a, y = 0$; $y = a, z = 0$ and $z = a, x = 0$. Prove that the equation of its locus is $xy + yz + zx - a(x + y + z - a) = 0$. [5]

12. Show that only one tangent plane can be drawn to the sphere $x^2 + y^2 + z^2 - 2x + 6y + 2z + 8 = 0$, [5] through the straight line $3x - 4y - 8 = 0 = y - 3z + 2$. Find the equation of the plane. [5]

GROUP – D

Answer **any two** from question no. 13 to 15 : (2 × 7.5)

13. A particle of mass m on a straight line is attracted towards the origin on it with a force $m\mu$ times the distance from it and the resistance to motion at any point is mk times the square of the velocity there. If it starts from rest at a distance 'a' from the origin, prove that it will come to rest again at a distance 'b' from the origin, where $(1 + 2ak)e^{-2ak} = (1 - 2bk)e^{2bk}$.

14. A heavy uniform chain of length $2l$, hangs over a small smooth fixed pulley, the length l+c being at one side and $l - c$ at the other. If the end of the shorter portion be held and then let go, show that the chain will slip off the pulley in time

$$\left(\frac{l}{g}\right)^{1/2} \log \frac{l + \sqrt{l^2 - c^2}}{c}, (l > c).$$

15. If h be the height attained by a particle when projected with a velocity v from the earth's surface supposing its attraction constant and H the corresponding height when the variation of gravity is taken into account, prove that

$$\frac{1}{h} - \frac{1}{H} = \frac{1}{R}$$,where R is the radius of the earth.

——— × ———